4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2 + 1}{2y}$.
   (a) Find the slope of the graph of f at the point where $x = 1$.
   (b) Write an equation for the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.
   (c) Find $f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2 + 1}{2y}$ with the initial condition $f(1) = 4$.
   (d) Use your solution from part (c) to find $f(1.2)$.

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1998 AP Calculus AB Scoring Guidelines

4. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f the slope is given by $\dfrac{3x^2 + 1}{2y}$.
   (a) Find the slope of the graph of f at the point where $x = 1$.
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   (d) Use your solution from part (c) to find $f(1.2)$.

(a) $\dfrac{dy}{dx} = \dfrac{3x^2 + 1}{2y}$

$\left.\dfrac{dy}{dx}\right|_{\substack{x=1\\y=4}} = \dfrac{3+1}{2\cdot 4} = \dfrac{4}{8} = \dfrac{1}{2}$

1: answer

(b) $y - 4 = \dfrac{1}{2}(x - 1)$

$f(1.2) - 4 \approx \dfrac{1}{2}(1.2 - 1)$

$f(1.2) \approx 0.1 + 4 = 4.1$

2
- 1: equation of tangent line
- 1: uses equation to approximate $f(1.2)$

(c) $2y\,dy = (3x^2 + 1)\,dx$

$\displaystyle\int 2y\,dy = \int (3x^2 + 1)\,dx$

$y^2 = x^3 + x + C$

$4^2 = 1 + 1 + C$

$14 = C$

$y^2 = x^3 + x + 14$

$y = \sqrt{x^3 + x + 14}$ is branch with point $(1, 4)$

$f(x) = \sqrt{x^3 + x + 14}$

5
- 1: separates variables
- 1: antiderivative of dy term
- 1: antiderivative of dx term
- 1: uses $y = 4$ when $x = 1$ to pick one function out of a family of functions
- 1: solves for y

  0/1 if solving a linear equation in y
  0/1 if no constant of integration

Note: max 0/5 if no separation of variables

Note: max 1/5 [1-0-0-0-0] if substitutes value(s) for x, y, or dy/dx before antidifferentiation

(d) $f(1.2) = \sqrt{1.2^3 + 1.2 + 14} \approx 4.114$

1: answer, from student's solution to the given differential equation in (c)

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